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Chapter 23

ELECTRIC FIELDS

In this chapter, we begin the study of electromagnetism. The first link that we will make to our previous study is through the concept of force. The electromagnetic force between charged particles is one of the fundamental forces of nature. We begin by describing some basic properties of one manifestation of the electromagnetic force, the electric force. We then discuss Coulomb’s law, which is the fundamental law governing the electric force between any two charged particles. Next, we introduce the concept of an electric field associated with a charge distribution and describe its effect on other charged particles. We then show how to use Coulomb’s law to calculate the electric field for a given charge distribution. The chapter concludes with a discussion of the motion of a charged particle in a uniform electric field.

23.1 Properties of Electric Charges

A number of simple experiments demonstrate the existence of electric forces. For example, after rubbing a balloon on your hair on a dry day, you will find that the balloon attracts bits of paper. The attractive force is often strong enough to suspend the paper from the balloon.

When materials behave in this way, they are said to be electrified or to have become electrically charged. In a series of simple experiments, it was found that there are two kinds of electric charges, which were given the names positive and negative by Benjamin Franklin (1706–1790). Electrons are identified as having negative charge, and protons are positively charged. Charges of the same sign repel one another and charges with opposite signs attract one another.

Another important aspect of electricity that arises from experimental observations is that electric charge is always conserved in an isolated system. That
is, when one object is rubbed against another, charge is not created in the process. The electrified state is due to a transfer of charge from one object to the other. One object gains some amount of negative charge while the other gains an equal amount of positive charge.

In 1909, Robert Millikan (1868–1953) discovered that electric charge always occurs as integral multiples of a fundamental amount of charge \( e \). In modern terms, the electric charge \( q \) is said to be quantized, where \( q \) is the standard symbol used for charge as a variable. That is, electric charge exists as discrete “packets,” and we can write

\[
q = \pm Ne
\]  

(23.1)

where \( N \) is some integer. Other experiments in the same period showed that the electron has a charge \(-e\) and the proton has a charge of equal magnitude but opposite sign \(+e\). Some particles, such as the neutron, have no charge.

**Quick Quiz 23.1.1**

When objects A and B are brought together, they repel. When objects B and C are brought together, they also repel. Which of the following are true?

A. Objects A and C possess charges of the same sign.
B. Objects A and C possess charges of opposite sign.
C. All three objects possess charges of the same sign.
D. One object is neutral.
E. Additional experiments must be performed to determine the signs of the charges.

**23.2 Charging Objects By Induction**

It is convenient to classify materials in terms of the ability of electrons to move through the material:

Electrical *conductors* are materials in which some of the electrons are free electrons that are not bound to atoms and can move relatively freely through the material; electrical *insulators* are materials in which all electrons are bound to atoms and cannot move freely through the material.

Materials such as glass, rubber, and dry wood fall into the category of electrical insulators. When such materials are charged by rubbing, only the area...
rubbed becomes charged and the charged particles are unable to move to other regions of the material.

In contrast, materials such as copper, aluminum, and silver are good electrical conductors. When such materials are charged in some small region, the charge readily distributes itself over the entire surface of the material.

*Semiconductors* are a third class of materials, and their electrical properties are somewhere between those of insulators and those of conductors. Silicon and germanium are well-known examples of semiconductors commonly used in the fabrication of a variety of electronic chips used in computers, cellular telephones, and home theater systems. The electrical properties of semiconductors can be changed over many orders of magnitude by the addition of controlled amounts of certain atoms to the materials.

Charging an object by induction requires no contact with the object inducing the charge. That is in contrast to charging an object by rubbing (that is, by conduction), which does require contact between the two objects.

![Figure 23.2](image)

A process similar to induction in conductors takes place in insulators. In most neutral molecules, the center of positive charge coincides with the center of negative charge. In the presence of a charged object, however, these centers inside each molecule in an insulator may shift slightly, resulting in more positive charge on one side of the molecule than on the other. This realignment of charge within
individual molecules produces a layer of charge on the surface of the insulator as shown in Figure 23.2a. The proximity of the positive charges on the surface of the object and the negative charges on the surface of the insulator results in an attractive force between the object and the insulator. Your knowledge of induction in insulators should help you explain why a charged rod attracts bits of electrically neutral paper as shown in Figure 23.2b.

**Quick Quiz 23.2.1**

When objects A and B are brought together, they attract. When objects B and C are brought together, they repel. Which of the following are necessarily true?

A. Objects A and C possess charges of the same sign.
B. Objects A and C possess charges of opposite sign.
C. All three objects possess charges of the same sign.
D. One object is neutral.
E. Additional experiments must be performed to determine information about the charges on the objects.

**23.3 Coulomb’s Law**

Charles Coulomb measured the magnitudes of the electric forces between charged objects using the torsion balance, which he invented (Fig. 23.3). The electric force between charged spheres A and B in Figure 23.3 causes the spheres to either attract or repel each other, and the resulting motion causes the suspended fiber to twist. Because the restoring torque of the twisted fiber is proportional to the angle through which the fiber rotates, a measurement of this angle provides a quantitative measure of the electric force of attraction or repulsion. Once the spheres are charged by rubbing, the electric force between them is very large compared with the gravitational attraction, and so the gravitational force can be neglected.

From Coulomb’s experiments, we can generalize the properties of the electric force (sometimes called the electrostatic force) between two stationary charged particles. We use the term point charge to refer to a charged particle of zero size. The electrical behavior of electrons and protons is very well described by modeling them as point charges. From experimental observations, we find that the magnitude of the electric force (sometimes called the Coulomb force) between two point charges is given by **Coulomb’s law:**
\[ F = k \frac{|q_1||q_2|}{r^2} \]  \hspace{1cm} (23.2)

where \( k \) is a constant called the Coulomb constant.

The value of the Coulomb constant depends on the choice of units. The SI unit of charge is the coulomb (C). The Coulomb constant \( k_e \) in SI units has the value:

\[ k = \frac{1}{4\pi\varepsilon_0} = 9.10^{9} \text{Nm}^2\text{C}^{-2} \]  \hspace{1cm} (23.3)

Where the constant \( \varepsilon_0 \) is known as the permittivity of free space and has the value:

\[ \varepsilon_0 = 8.85.10^{-12}(F/m) \]

The smallest unit of free charge \( e \) known in nature, the charge on an electron (-e) or a proton (+e), has a magnitude

\[ e = 1.6.10^{-19} \text{C} \]

When dealing with Coulomb’s law, remember that force is a vector quantity and must be treated accordingly. Coulomb’s law expressed in vector form for the electric force exerted by a charge \( q_1 \) on a second charge \( q_2 \), written \( \vec{F}_{12} \), is:

\[ Ph.D. Nguyen Thi Ngoc Nu \]
\[ \vec{F}_{12} = k \frac{q_1 q_2}{r^2} \hat{r}_{12} \quad (23.4) \]

where \( \hat{r}_{12} \) is a unit vector directed from \( q_1 \) toward \( q_2 \) as shown in Figure 23.4. Because the electric force obeys Newton’s third law, the electric force exerted by \( q_2 \) on \( q_1 \) is equal in magnitude to the force exerted by \( q_1 \) on \( q_2 \) and in the opposite direction.

When more than two charges are present, the force between any pair of them is given by Equation 23.4. Therefore, the resultant force on any one of them equals the vector sum of the forces exerted by the other individual charges:

\[ \vec{F}_0 = \vec{F}_{10} + \vec{F}_{20} + \ldots + \vec{F}_{n0} = \sum_{i=1}^{n} \vec{F}_{i0} \quad (23.5) \]
Quick Quiz 23.3.1

Object A has a charge of +2 \( \mu \text{C} \), and object B has a charge of +6 \( \mu \text{C} \). Which statement is true about the electric forces on the objects?

A. \( F_{AB} = -3F_{BA} \)  
B. \( F_{AB} = -F_{BA} \)  
C. \( 3F_{AB} = -F_{BA} \)  
D. \( F_{AB} = 3F_{BA} \)  
E. \( F_{AB} = F_{BA} \)  
F. \( 3F_{AB} = F_{BA} \)

Quick Quiz 23.3.2

Two point charges attract each other with an electric force of magnitude F. If the charge on one of the particles is reduced to one-third its original value and the distance between the particles is doubled, what is the resulting magnitude of the electric force between them?

A. \( \frac{1}{12} F \)  
B. \( \frac{1}{3} F \)  
C. \( \frac{1}{6} F \)  
D. \( \frac{3}{4} F \)  
E. \( \frac{3}{2} F \)

Example 23.3.1

Consider three point charges located at the corners of a right triangle as shown in the Figure P23.3.1, where \( q_1 = q_3 = 5.00 \mu \text{C}, q_2 = 2.00 \mu \text{C}, \) and \( a = 0.100 \text{ m} \). Find the resultant force exerted on \( q_3 \).

Problem 23.3.1

Three point charges lie along a straight line as shown in Figure P23.3.1, where \( q_1 = 6.00 \mu \text{C}, q_2 = 1.50 \mu \text{C} \) and \( q_3 = -2.00 \mu \text{C} \). The separation distances are
d_1 = 3.00 cm and d_2 = 2.00 cm. Calculate the magnitude and direction of the net electric force on
a) q_1,
b) q_2,
c) q_3.

Problem 23.3.2

Three charged particles are located at the corners of an equilateral triangle as shown in Figure P23.3.2. Calculate the total electric force on the 7.00-µC charge.

23.4 Particle in an Electric Field

The concept of a field was developed by Michael Faraday (1791–1867) in the context of electric forces and is of such practical value that we shall devote much attention to it in the next several chapters. In this approach, an electric field is said to exist in the region of space around a charged object, the source charge. The presence of the electric field can be detected by placing a test charge in the field and noting the electric force on it. We define the electric field due to the source charge at the location of the test charge to be the electric force on the test charge per unit charge, or, to be more specific, the electric field vector at a point in space is defined as the electric force acting on a positive test charge placed at that point divided by the test charge:
The vector $\mathbf{E}$ has the SI units of newtons per coulomb (N/C). The direction of $\mathbf{E}$ as shown in Figure 23.6 is the direction of the force a positive test charge experiences when placed in the field. Note that $\mathbf{E}$ is the field produced by some charge or charge distribution separate from the test charge; it is not the field produced by the test charge itself. Also note that the existence of an electric field is a property of its source; the presence of the test charge is not necessary for the field to exist. The test charge serves as a detector of the electric field: an electric field exists at a point if a test charge at that point experiences an electric force.

If an arbitrary charge $q$ is placed in an electric field $\mathbf{E}$, it experiences an electric force given by:

$$\mathbf{F} = q\mathbf{E}$$

(23.7)

To determine the direction of an electric field, consider a point charge $q$ as a source charge. This charge creates an electric field at all points in space surrounding it. A test charge $q_0$ is placed at point P, a distance $r$ from the source charge, as in Figure 23.7a. We imagine using the test charge to determine the direction of the electric force and therefore that of the electric field. According to Coulomb’s law, the force exerted by $q$ on the test charge is

$$\mathbf{F}_{12} = k \frac{q_1 q_2}{r^2} \mathbf{r}_{12}$$
This force in Figure 23.7a is directed away from the source charge $q$.

Because the electric field at $P$, the position of the test charge, is defined by $\vec{E} = \frac{q}{q_0} \vec{F}$, the electric field at $P$ created by $q$ is:

$$\vec{E} = k \frac{q}{r^2} \hat{r}$$

$$E = k \frac{|q|}{r^2} \quad (23.8)$$

If the source charge $q$ is positive, Figure 23.7b shows the situation with the test charge removed: the source charge sets up an electric field at $P$, directed away from $q$. If $q$ is negative as in Figure 23.7c, the force on the test charge is toward the source charge, so the electric field at $P$ is directed toward the source charge as in Figure 23.7d.

To calculate the electric field at a point $P$ due to a small number of point charges, we first calculate the electric field vectors at $P$ individually using Equation 23.8 and then add them vectorially. In other words, at any point $P$, the total electric field due to a group of source charges equals the vector sum of the electric fields of all the charges. This superposition principle applied to fields follows directly from the vector addition of electric forces.
Quick Quiz 23.4.1

A test charge of $+3\mu\text{C}$ is at a point P where an external electric field is directed to the right and has a magnitude of $4.10^6 \text{ N/C}$. If the test charge is replaced with another test charge of $-3\text{mC}$, the external electric field at P

A. is unaffected  
B. reverses direction  
C. changes in a way that cannot be determined.

Quick Quiz 23.4.2

An object with negative charge is placed in a region of space where the electric field is directed vertically upward. What is the direction of the electric force exerted on this charge?

A. It is up.  
B. It is down.  
C. There is no force.  
D. The force can be in any direction.

Quick Quiz 23.4.3

Estimate the magnitude of the electric field due to the proton in a hydrogen atom at a distance of $5.29 \times 10^{-11} \text{ m}$, the expected position of the electron in the atom.

A. $10^{-11} \text{ N/C}$  
B. $10^8 \text{ N/C}$  
C. $10^{14} \text{ N/C}$  
D. $10^6 \text{ N/C}$  
E. $10^{12} \text{ N/C}$

Problem 23.4.1

Four charged particles are at the corners of a square of side $a$ as shown in Figure P23.4.1. Determine

a) the electric field at the location of charge $q$.

b) the total electric force exerted on $q$.

![Figure P23.4.1]
Problem 23.4.2

Three charges are at the corners of an equilateral triangle as shown in Figure P23.4.2.

![Figure P23.4.2](image)

a) Calculate the electric field at the position of the 2.00 \( \mu \text{C} \) charge due to the 7.00 \( \mu \text{C} \) and -4.00 \( \mu \text{C} \) charges.

b) Use your answer to part a) to determine the force on the 2.00 \( \mu \text{C} \) charge.

Problem 23.4.3

In Figure P23.4.3, determine the point (other than infinity) at which the electric field is zero.

![Figure P23.4.3](image)

23.5 Electric Field of a Continuous Charge Distribution

To set up the process for evaluating the electric field created by a continuous charge distribution, let’s use the following procedure. First, divide the charge distribution into small elements, each of which contains a small charge \( \Delta q \) as shown in Figure 23.8. Next, use Equation 23.9 to calculate the electric field due to one of these elements at a point P. Finally, evaluate the total electric field at P due to the charge distribution by summing the contributions of all the charge elements (that is, by applying the superposition principle).

The electric field at P due to one charge element carrying charge \( \Delta q \) is

\[
\vec{\Delta E} = k \frac{\Delta q}{r^2} \hat{r}
\]
The total electric field at P due to all elements in the charge distribution is approximately

\[ \vec{E} = k \sum \frac{\Delta q_i}{r_i^2} \hat{r}_i \]

where the index \( i \) refers to the \( i \)th element in the distribution. Because the number of elements is very large and the charge distribution is modeled as continuous, the total field at P is:

\[ \vec{E} = \int k \frac{dq}{r^2} \hat{r} \quad (23.9) \]

Let’s illustrate this type of calculation with several examples in which the charge is distributed on a line, on a surface, or throughout a volume. When performing such calculations, it is convenient to use the concept of a charge density along with the following notations:

- If a charge \( Q \) is uniformly distributed throughout a volume \( V \), the volume charge density \( \rho \) is defined by \( \rho = q/V \), where \( r \) has units of coulombs per cubic meter (C/m\(^3\)).
- If a charge \( Q \) is uniformly distributed on a surface of area \( A \), the surface charge density \( \sigma \) is defined by \( \sigma = q/S \), where \( s \) has units of coulombs per square meter (C/m\(^2\)).
- If a charge \( Q \) is uniformly distributed along a line of length, the linear
**charge density** \( \lambda \) is defined by \( \lambda = \frac{q}{l} \), where \( l \) has units of coulombs per meter (C/m).

If the charge is nonuniformly distributed over a volume, surface, or line, the amounts of charge \( dq \) in a small volume, surface, or length element are:

\[
dq = \lambda \, dl = \sigma \, dS = \rho \, dV
\]

**Example 23.5.1**

A ring of radius \( a \) carries a uniformly distributed positive total charge \( Q \). Calculate the electric field due to the ring at a point \( P \) lying a distance \( x \) from its center along the central axis perpendicular to the plane of the ring (Figure E23.5.1).

**23.6 Electric Field Lines**

We have defined the electric field in the mathematical representation with Equation 23.6. Let’s now explore a means of visualizing the electric field in a pictorial representation. A convenient way of visualizing electric field patterns is to draw lines, called electric field lines and first introduced by Faraday, that are related to the electric field in a region of space in the following manner:

- The electric field vector is tangent to the electric field line at each point. Direction of the line is the same as that the electric field vector (Figure 23.9).

- The field lines are close together where the electric field is strong and far apart where the field is weak (Figure 23.10).
The rules for drawing electric field lines are as follows:

- The lines must begin on a positive charge and terminate on a negative charge. In the case of an excess of one type of charge, some lines will begin or end infinitely far away (Figure 23.11).

- The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.
- No two field lines can cross.

Quick Quiz 23.6.1

Rank the magnitudes of the electric field at points A, B, and C shown in Figure Q23.6.1 (greatest magnitude first).
Problem 23.6.1

Figure P23.6.1 shows the electric field lines for two point charges separated by a small distance.

(a) Determine the ratio $q_1/q_2$.

(b) What are the signs of $q_1$ and $q_2$?

23.7 Motion of Charged Particles in a Uniform Electric Field

When a particle of charge $q$ and mass $m$ is placed in an electric field $\vec{E}$, the electric force exerted on the charge is $q\vec{E}$. If that is the only force exerted on the particle, it must be the net force, and it causes the particle to accelerate according to the particle under a net force model. Therefore,

$$\vec{F} = q\vec{E} = ma$$

and the acceleration of the particle is

$$a = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m} \quad (23.10)$$
If $\vec{E}$ is uniform (that is, constant in magnitude and direction), and the particle is free to move, the electric force on the particle is constant and we can apply the particle under constant acceleration model to the motion of the particle. Therefore, the particle in this situation is described by three analysis models: particle in a field, particle under a net force, and particle under constant acceleration! If the particle has a positive charge, its acceleration is in the direction of the electric field. If the particle has a negative charge, its acceleration is in the direction opposite the electric field.
Chapter 24
GAUSS’S LAW

In Chapter 23, we showed how to calculate the electric field due to a given charge distribution by integrating over the distribution. In this chapter, we describe Gauss’s law and an alternative procedure for calculating electric fields. Gauss’s law is based on the inversesquare behavior of the electric force between point charges. Although Gauss’s law is a direct consequence of Coulomb’s law, it is more convenient for calculating the electric fields of highly symmetric charge distributions and makes it possible to deal with complicated problems using qualitative reasoning. As we show in this chapter, Gauss’s law is important in understanding and verifying the properties of conductors in electrostatic equilibrium.

24.1 Electric Flux

Consider an electric field that is uniform in both magnitude and direction as shown in Figure 24.1. The field lines penetrate a rectangular surface of area whose plane is oriented perpendicular to the field. Recall from Section 23.6 that the number of lines per unit area (in other words, the line density) is proportional to the magnitude of the electric field. Therefore, the total number of lines penetrating the surface is proportional to the product EA. This product of the magnitude of the electric field and surface area perpendicular to the field is called the electric flux $\Phi_E$

$$\Phi_E = EA$$ (24.1)

![Figure 24.1](image-url)
From the SI units of $E$ and $A$, we see that $\Phi_e$ has units of newton meters squared per coulomb ($\text{N.m}^2/\text{C}$). Electric flux is proportional to the number of electric field lines penetrating some surface.

If the electric field is uniform and makes an angle $\theta$ with the normal to a surface of area $A$, the electric flux through the surface is:

$$\Phi_e = EA \cos \theta$$  \hspace{1cm} (24.2)

In general, the electric flux through a surface is

$$\Phi_e = \int E\cdot dA \cdot \cos \theta$$

**Quick Quiz 24.1.1**

Suppose a point charge is located at the center of a spherical surface. Now the radius of the sphere is halved. What happens to the flux through the sphere and the magnitude of the electric field at the surface of the sphere?

A. The flux and field both increase.
B. The flux and field both decrease.
C. The flux increases, and the field decreases.
D. The flux decreases, and the field increases.
E. The flux remains the same, and the field increases.
F. The flux decreases, and the field remains the same.

**24.2 Gauss’s Law**

In this section, we describe a general relationship between the net electric flux through a closed surface (often called a gaussian surface) and the charge enclosed by the surface. This relationship, known as Gauss’s law, is of fundamental importance in the study of electric fields.

Consider a positive point charge $q$ located at the center of a sphere of radius $r$ as shown in Figure 24.3. We know that the magnitude of the electric field
everywhere on the surface of the sphere is \( \mathbf{E} = \frac{kq}{r^2} \). The field lines are directed radially outward and hence are perpendicular to the surface at every point on the surface. That is, at each surface point, \( \mathbf{E} \) is parallel to the vector \( \Delta \mathbf{A}_i \) representing a local element of area \( \Delta A_i \) surrounding the surface point. Therefore,

\[
\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \oint \mathbf{E} dA = EA = k \frac{q}{r^2} 4\pi r^2 = 4\pi kq
\]

\[
\Phi_E = \frac{q}{\varepsilon_0} \quad (24.3)
\]

Figure 24.3

Now consider several closed surfaces surrounding a charge \( q \) as shown in Figure 24.4. Surface \( S_1 \) is spherical, but surfaces \( S_2 \) and \( S_3 \) are not. From Equation 24.5, the flux that passes through \( S_1 \) has the value \( \frac{q}{\varepsilon_0} \). As discussed in the preceding section, flux is proportional to the number of electric field lines passing through a surface. The construction shown in Figure 24.4 shows that the number of lines through \( S_1 \) is equal to the number of lines through the nonspherical surfaces \( S_2 \) and \( S_3 \).
The net electric flux $F_E$ through any closed surface (gaussian surface) is equal to the net charge $q_{in}$ inside the surface divided by $\varepsilon_0$:

$$\Phi_E = \oint E \cdot dA = \frac{q_{in}}{\varepsilon_0} \quad (24.4)$$

Quick Quiz 24.2.1

If the net flux through a gaussian surface is zero, the following four statements could be true. Which of the statements must be true?

A. There are no charges inside the surface.
B. The net charge inside the surface is zero.
C. The electric field is zero everywhere on the surface.
D. The number of electric field lines entering the surface equals the number leaving the surface.

Quick Quiz 24.2.2

Consider the charge distribution shown in Figure Q.24.2.2. The charges contributing to the total electric flux through surface S’ are

A. $q_1$ only
B. $q_4$ only
C. $q_2$ and $q_3$
D. all four charges
E. none of the charges.
Question 24.2.1
A spherical gaussian surface surrounds a point charge $q$. Describe what happens to the total flux through the surface if
a) the charge is tripled,
b) the radius of the sphere is doubled,
c) the surface is changed to a cube,
d) the charge is moved to another location inside the surface.

Problem 24.2.1
The following charges are located inside a submarine: 5.00 $\mu$C, -9.00 $\mu$C, 27.0 $\mu$C, and -84.0 $\mu$C. Calculate the net electric flux through the hull of the submarine.

24.3 Application of Gauss’s Law to Various Charge Distributions
As mentioned earlier, Gauss’s law is useful for determining electric fields when the charge distribution is highly symmetric. The following examples demonstrate ways of choosing the gaussian surface over which the surface integral given by Equation 24.4 can be simplified and the electric field determined. In choosing the surface, always take advantage of the symmetry of the charge distribution so that $E$ can be removed from the integral. The goal in this type of calculation is to determine a surface for which each portion of the surface satisfies one or more of the following conditions:

1. The value of the electric field can be argued by symmetry to be constant over the portion of the surface.
2. The dot product in Equation 24.4 can be expressed as a simple algebraic product $E \, dA$ because $\vec{E}$ and $d\vec{A}$ are parallel.
3. The dot product in Equation 24.4 is zero because $\vec{E}$ and $d\vec{A}$ are perpendicular.
4. The electric field is zero over the portion of the surface.
Example

Find the electric field due to an infinite plane of positive charge with uniform surface charge density $\sigma$.

Solution

By symmetry, $\vec{E}$ must be perpendicular to the plane at all points. The direction of $\vec{E}$ is away from positive charges, indicating that the direction of $\vec{E}$ on one side of the plane must be opposite its direction on the other side as shown in Figure 24.5. A gaussian surface that reflects the symmetry is a small cylinder whose axis is perpendicular to the plane and whose ends each have an area $A$ and are equidistant from the plane. Because $\vec{E}$ is parallel to the curved surface of the cylinder—and therefore perpendicular to $dA$ at all points on this surface-condition (3) is satisfied and there is no contribution to the surface integral from this surface.

For the flat ends of the cylinder, conditions (1) and (2) are satisfied. The flux through each end of the cylinder is $EA$; hence, the total flux through the entire gaussian surface is just that through the ends, $\Phi_E = 2EA$.

Write Gauss’s law for this surface, noting that the enclosed charge is $q_in = \sigma A$.

$$\Phi_E = \frac{q_in}{\epsilon_0}$$

Solve for $E$:

$$E = \frac{\sigma}{2\epsilon_0} \quad (24.5)$$

Because the distance from each flat end of the cylinder to the plane does not appear in Equation 24.5, we conclude that $E = \frac{\sigma}{2\epsilon_0}$ at any distance from the plane. That is,
the field is uniform everywhere. Figure 24.6 shows this uniform field due to an infinite plane of charge, seen edge-on.

![Figure 24.6](image)

**24.4 Conductors in Electrostatic Equilibrium**

As we learned in Section 23.2, a good electrical conductor contains charges (electrons) that are not bound to any atom and therefore are free to move about within the material. When there is no net motion of charge within a conductor, the conductor is in electrostatic equilibrium. A conductor in electrostatic equilibrium has the following properties:

- The electric field is zero everywhere inside the conductor.
- If an isolated conductor carries a charge, the charge resides on its surface.
- The electric field just outside the conductor is perpendicular to its surface and has a magnitude: \( E = \frac{\sigma}{\varepsilon_0} \).
- On an irregularly shaped conductor, the surface charge density is greatest where the radius of curvature of the surface is the smallest.

**Problem 24.4.1**

A long, straight metal rod has a radius of 5.00 cm and a charge per unit length of 30.0 nC/m. Find the electric field

a) 3.00 cm and

b) 10.00 cm from the axis of the rod, where distances are measured perpendicular to the rod.
Chapter 25

ELECTRIC POTENTIAL

In Chapter 23, we linked our new study of electromagnetism to our earlier studies of force. Now we make a new link to our earlier investigations into energy.

25.1 Electric Potential and Potential Difference

When a charge \( q \) is placed in an electric field \( \overrightarrow{E} \) created by some source charge distribution, the particle in a field model tells us that there is an electric force \( q\overrightarrow{E} \) acting on the charge. This force is conservative because the force between charges described by Coulomb’s law is conservative. Let us identify the charge and the field as a system. If the charge is free to move, it will do so in response to the electric force. Therefore, the electric field will be doing work on the charge. This work is internal to the system.

![Figure 25.1](image)

The work done by the electric field:

\[
W_{AB} = \int_{A}^{B} \overrightarrow{F} \cdot d\overrightarrow{s} = \int_{A}^{B} q\overrightarrow{E} \cdot d\overrightarrow{s} = U_{A} - U_{B}
\]

For a given position of the charge in the field, the charge–field system has a potential energy \( U \). Dividing the potential energy by the charge gives a physical quantity that depends only on the source charge distribution and has a value at every point in an electric field. This quantity is called the electric potential (or simply the potential) \( V \):
Because potential energy is a scalar quantity, electric potential also is a scalar quantity.

The potential difference $\Delta V = V_A - V_B$ between two points $A$ and $B$ in an electric field is defined as the change in electric potential energy of the system when a charge $q$ is moved between the points divided by the charge:

$$\Delta V = V_A - V_B = \frac{U_A - U_B}{q} = \int_A^B E \cdot d\vec{s}$$

(25.2)

Because electric potential is a measure of potential energy per unit charge, the SI unit of both electric potential and potential difference is joules per coulomb, which is defined as a volt (V):

$$1 \text{ V} = 1 \text{ J/C}$$

Quick Quiz 25.1.1

In Figure Q25.1.1, two points $A$ and $B$ are located within a region in which there is an electric field. The potential difference $\Delta V = V_A - V_B$ is

A. positive
B. negative
C. zero.

Quick Quiz 25.1.1

In Figure, a negative charge is placed at $A$ and then moved to $B$. The change in potential energy of the charge–field system for this process is

A. positive
B. negative  
C. zero.

25.2 Potential Differences in a Uniform Electric Field

Equations 25.2 hold in all electric fields, whether uniform or varying, but they can be simplified for the special case of a uniform field. First, consider a uniform electric field directed along the negative y-axis as shown in Figure 25.2. Let’s calculate the potential difference between two points A and B separated by a distance d, where the displacement \( \vec{s} \) points from A toward B and is parallel to the field lines. Equation 25.2 gives:

\[
\Delta V = V_A - V_B = \int_A^B \vec{E}.d\vec{s} = \int_A^B E ds
\]

Because \( E \) is constant, it can be removed from the integral sign, which gives

\[
\Delta V = Ed \quad \text{(25.3)}
\]

Electric field lines always point in the direction of decreasing electric potential.

Now suppose a charge q moves from A to B. We can calculate the change in the potential energy of the charge-field system from Equations 25.1 and 25.3:

\[
\Delta U = q\Delta V = qEd
\]

Now consider the more general case of a charged particle that moves between A and B in a uniform electric field such that the vector \( \vec{s} \) is not parallel to the field lines as shown in Figure 25.3. In this case, Equation 25.2 gives
\[ \Delta V = \int_{A}^{B} \vec{E} \cdot d\vec{s} = \vec{E} \cdot \vec{s} = Es \cos \theta \]

Finally, we conclude from Equation 25.8 that all points in a plane perpendicular to a uniform electric field are at the same electric potential. The name equipotential surface is given to any surface consisting of a continuous distribution of points having the same electric potential.

**Quick Quiz 25.2.1**

The labeled points in Figure Q25.2.1 are on a series of equipotential surfaces associated with an electric field. Rank (from greatest to least) the work done by the electric field on a positively charged particle that moves from A to B; from B to C; from C to D; from D to E.

**25.3 Electric Potential and Potential Energy Due to Point Charges**

As discussed in Section 23.4, an isolated positive point charge qproduces an electric field directed radially outward from the charge. To find the electric potential at a point located a distance r from the charge, let’s begin with the general expression for potential difference:

\[ V_{A} - V_{B} = \int_{A}^{B} \vec{E} \cdot d\vec{s} \]
At any point in space, the electric field due to the point charge is

\[ \mathbf{E} = k \frac{q}{r^2} \mathbf{r} \]

Therefore,

\[ V_A - V_B = \int_A^B k \frac{q}{r^2} \mathbf{r} \cdot d\mathbf{s} = \int_A^B k \frac{q}{r^2} \mathbf{r} \cdot d\mathbf{r} = k \int_A^B \frac{dr}{r^2} \]

\[ V_A - V_B = \frac{kq}{r_A} - \frac{kq}{r_B} \quad (25.4) \]

Equation 25.4 expresses the important result that the potential difference between any two points A and B in a field created by a point charge depends only on the radial coordinates \( r_A \) and \( r_B \). It is customary to choose the reference of electric potential for a point charge to be \( V = 0 \) at \( r = \infty \). With this reference choice, the electric potential due to a point charge at any distance \( r \) from the charge is

\[ V = k \frac{q}{r} \quad (25.5) \]

We obtain the electric potential resulting from two or more point charges by applying the superposition principle. That is, the total electric potential at some point \( P \) due to several point charges is the sum of the potentials due to the individual charges. For a group of point charges, we can write the total electric potential at \( P \) as

\[ V = \sum_i V_i = \sum_i k \frac{q_i}{r_i} \quad (25.6) \]

**Quick Quiz 25.3.1**

A spherical balloon contains a positively charged object at its center. As the balloon is inflated to a greater volume while the charged object remains at the center, does the electric potential at the surface of the balloon

A. increase
B. decrease,
C. remain the same?
Quick Quiz 25.3.2

In Figure Q25.3.2, take $q_1$ to be a negative source charge and $q_2$ to be the test charge. If $q_2$ is initially positive and is changed to a charge of the same magnitude but negative, the potential at the position of $q_2$ due to $q_1$

A. increases
B. decreases
C. remains the same.

![Figure Q25.3.2](image)

Problem 25.3.1

Given two 2.00 μC charges, as shown in Figure P25.3.1, and a positive test charge $q = 1.28 \times 10^{-18}$ C at the origin. What is the electric potential at the origin due to the two 2.00 mC charges?

![Figure P25.3.1](image)

25.4 Obtaining the Value of the Electric Field from the Electric Potential

The electric field $\vec{E}$ and the electric potential $V$ are related as shown in Equation 25.2, which tells us how to find $\Delta V$ if the electric field $\vec{E}$ is known. What if the situation is reversed? How do we calculate the value of the electric field if the electric potential is known in a certain region?

From Equation 25.2, the potential difference $dV$ between two points a distance $ds$ apart can be expressed as:

$$V_A - V_B = \int_{A}^{B} \vec{E} \cdot d\vec{s}$$

$$dV = -\vec{E} \cdot d\vec{s}$$ (25.7)
If the electric field has only one component \( E_x \), then \( \vec{E} \cdot d\vec{s} = E_x dx \). Therefore,

\[
E_x = -\frac{dV}{dx} \tag{25.8}
\]

Imagine starting at a point and then moving through a displacement \( d\vec{s} \) along an equipotential surface. For this motion, \( dV=0 \) because the potential is constant along an equipotential surface. From Equation 25.8, we see that \( dV = -\vec{E} \cdot d\vec{s} = 0 \); therefore, because the dot product is zero, \( \vec{E} \) must be perpendicular to the displacement along the equipotential surface. This result shows that the *equipotential surfaces must always be perpendicular to the electric field lines passing through them* (Figure 25.4).

![Figure 25.4](image)

If the charge distribution creating an electric field has spherical symmetry such that the volume charge density depends only on the radial distance \( r \), the electric field is radial. In this case, \( \vec{E} \cdot d\vec{s} = E_r dr \). Therefore:

\[
E_r = -\frac{dV}{dr} \tag{25.9}
\]

**Quick Quiz 25.4.1**

In a certain region of space, the electric potential is zero everywhere along the \( x \) axis. From this we can conclude that the \( x \) component of the electric field in this region is

A. zero
B. in the +x direction
C. in the -x direction.
Quick Quiz 25.4.2

In a certain region of space, the electric field is zero. From this we can conclude that the electric potential in this region is
A. zero
B. Constant
C. positive
D. negative.

Quick Quiz 25.4.3

Consider the equipotential surfaces shown in Figure Q25.4.3. In this region of space, what is the approximate direction of the electric field?
A. It is out of the page.
B. It is into the page.
C. It is toward the top of the page.
D. It is toward the bottom of the page.
E. The field is zero.

![Figure Q25.4.3](image-url)

Quick Quiz 25.4.4

The electric potential at $x = 3.00 \text{ m}$ is 120 V, and the electric potential at $x=5.00 \text{ m}$ is 190 V. What is the $x$ component of the electric field in this region, assuming the field is uniform?
A. 140 N/C
B. 2140 N/C
C. 35.0 N/C
D. 235.0 N/C
E. 75.0 N/C.
Quick Quiz 25.4.5

In a certain region of space, a uniform electric field is in the x direction. A particle with negative charge is carried from x = 20.0 cm to x = 60.0 cm. Has the particle moved to a position where the electric potential is
A. higher than before,
B. unchanged,
C. lower than before,
D. predictable?

25.5 Electric Potential Due to Continuous Charge Distributions

In Section 25.3, we found how to determine the electric potential due to a small number of charges. What if we wish to find the potential due to a continuous distribution of charge? The electric potential in this situation can be calculated using two different methods. The first method is as follows. If the charge distribution is known, we consider the potential due to a small charge element dq, treating this element as a point charge (Fig. 25.5). From Equation 25.5, the electric potential dV at some point P due to the charge element dq is

\[ dV = \frac{k dq}{r} \quad (25.10) \]

where r is the distance from the charge element to point P. To obtain the total potential at point P, we integrate Equation 25.19 to include contributions from all elements of the charge distribution. Because each element is, in general, a different distance from point P and k is constant, we can express V as

\[ V = \int dV = k \int \frac{dq}{r} \quad (25.11) \]
Example 25.5.1

Find an expression for the electric potential at a point P located on the perpendicular central axis of a uniformly charged ring of radius \( a \) and total charge \( Q \).

Solution

We take point P to be at a distance \( x \) from the center of the ring as shown in Figure E25.5.1.

![Figure E25.5.1](image)

Use Equation 25.11 to express \( V \) in terms of the geometry:

\[
V_M = \oint dV = \oint \frac{k dq}{r} = \frac{k}{r} \oint dq
\]

\[
V_M = \frac{k Q}{\sqrt{a^2 + x^2}}
\]

\[
V_o = \frac{k Q}{a}
\]

25.6 Electric Potential Due to a Charged Conductor

Consider two points A and B on the surface of a charged conductor as shown in Figure 25.6. Along a surface path connecting these points, \( \vec{E} \) is always perpendicular to the displacement \( d\vec{s} \); therefore, \( \vec{E}.d\vec{s} = 0 \). Using this result and equation 25.2, we conclude that the potential difference between A and B is necessarily zero:

\[
V_A - V_B = \int_A^B \vec{E}.d\vec{s} = 0
\]

This result applies to any two points on the surface. Therefore, \( V \) is constant everywhere on the surface of a charged conductor in equilibrium. That is, the surface of any charged conductor in electrostatic equilibrium is an *equipotential surface*: every point on the surface of a charged conductor in equilibrium is at the
same electric potential. Furthermore, because the electric field is zero inside the conductor, the electric potential is constant everywhere inside the conductor and equal to its value at the surface.
Chapter 26
CAPACITANCE AND DIELECTRICS

In this chapter, we introduce the first of three simple circuit elements that can be connected with wires to form an electric circuit. Electric circuits are the basis for the vast majority of the devices used in our society. Here we shall discuss capacitors, devices that store electric charge. Capacitors are commonly used in a variety of electric circuits. For instance, they are used to tune the frequency of radio receivers, as filters in power supplies, to eliminate sparking in automobile ignition systems, and as energy-storing devices in electronic flash units.

26.1 Definition of Capacitance

Consider two conductors as shown in Figure 26.1. Such a combination of two conductors is called a capacitor. The conductors are called plates. If the conductors carry charges of equal magnitude and opposite sign, a potential difference \( dV \) exists between them.

What determines how much charge is on the plates of a capacitor for a given voltage? Experiments show that the quantity of charge \( Q \) on a capacitor is linearly proportional to the potential difference between the conductors; that is, \( Q \sim dV \). The proportionality constant depends on the shape and separation of the conductors. This relationship can be written as \( Q = C \Delta V \) if we define capacitance as follows:

The capacitance \( C \) of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors:

\[
Q = C \Delta V
\]
By definition capacitance is always a positive quantity. Furthermore, the charge $Q$ and the potential difference $\Delta V$ are always expressed in Equation 26.1 as positive quantities.

From Equation 26.1, we see that capacitance has SI units of coulombs per volt. Named in honor of Michael Faraday, the SI unit of capacitance is the farad (F):

$$1 \text{ F} = 1 \text{ C/V}$$

The farad is a very large unit of capacitance. In practice, typical devices have capacitances ranging from microfarads ($1 \mu\text{F}=10^{-6} \text{ F}$) to picofarads ($1 \text{pF}=10^{-12}\text{ F}$).

**Quick Quiz 26.1.1**

A capacitor stores charge $Q$ at a potential difference $\Delta V$. If the voltage applied by a battery to the capacitor is doubled to $2\Delta V$,

A. the capacitance falls to half its initial value and the charge remains the same
B. the capacitance and the charge both fall to half their initial values
C. the capacitance and the charge both double
D. the capacitance remains the same and the charge doubles.

**Quick Quiz 26.1.2**

What happens to the magnitude of the charge on each plate of a capacitor if the potential difference between the conductors is doubled?

A. It becomes four times larger.
B. It becomes two times larger.
C. It is unchanged.
D. It becomes one-half as large.
E. It becomes one-fourth as large.

**26.2 Calculating Capacitance**

Although the most common situation is that of two conductors, a single conductor also has a capacitance. For example, imagine a single spherical, charged conductor. The electric field lines around this conductor are exactly the same as if there were a conducting, spherical shell of infinite radius, concentric with the sphere and carrying a charge of the same magnitude but opposite sign. Therefore, we can identify the imaginary shell as the second conductor of a two-conductor capacitor.
The electric potential of the sphere of radius $a$ is simply $kQ/a$, and setting $V=0$ for the infinitely large shell gives

$$C = \frac{Q}{\Delta V} = \frac{R}{k} \quad (26.2)$$

This expression shows that the capacitance of an isolated, charged sphere is proportional to its radius and is independent of both the charge on the sphere and its potential, as is the case with all capacitors. Equation 26.1 is the general definition of capacitance in terms of electrical parameters, but the capacitance of a given capacitor will depend only on the geometry of the plates.

**Parallel-Plate Capacitors**

![Parallel-Plate Capacitors](image)

Figure 26.2

Two parallel, metallic plates of equal area $A$ are separated by a distance $d$ as shown in Figure 26.2. One plate carries a charge $+Q$, and the other carries a charge $-Q$. The surface charge density on each plate is $\sigma = Q/A$. If the plates are very close together (in comparison with their length and width), we can assume the electric field is uniform between the plates and zero elsewhere. The value of the electric field between the plates is:

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

Because the field between the plates is uniform, the magnitude of the potential difference between the plates equals $Ed$; therefore,

$$\Delta V = Ed = \frac{Qd}{\varepsilon_0 A}$$

Substituting this result into Equation 26.1, we find that the capacitance is

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That is, the capacitance of a parallel-plate capacitor is proportional to the area of its plates and inversely proportional to the plate separation.

Quick Quiz 26.2.1

Many computer keyboard buttons are constructed of capacitors, as shown in Figure Q26.2.1. When a key is pushed down, the soft insulator between the movable plate and the fixed plate is compressed. When the key is pressed, the capacitance

A. increases
B. decreases,
C. changes in a way that we cannot determine because the complicated electric circuit connected to the keyboard button may cause a change in $V$.

Problem 26.2.1

An isolated charged conducting sphere of radius 12.0 cm creates an electric field of $4.90 \times 10^4$ N/C at a distance 21.0 cm from its center.

a) What is its surface charge density?

b) What is its capacitance?

Problem 26.2.2

An air-filled capacitor consists of two parallel plates, each with an area of $7.60$ cm$^2$, separated by a distance of 1.80 mm. A 20.0-V potential difference is applied to these plates. Calculate

a) the electric field between the plates,

b) the capacitance,

c) the charge on each plate.
26.3 Combinations of Capacitors

In studying electric circuits, we use a simplified pictorial representation called a circuit diagram. Such a diagram uses circuit symbols to represent various circuit elements. The circuit symbols are connected by straight lines that represent the wires between the circuit elements. The circuit symbols for capacitors, batteries, and switches as well as the color codes used for them in this text are given in Figure 26.3. The symbol for the capacitor reflects the geometry of the most common model for a capacitor, a pair of parallel plates. The positive terminal of the battery is at the higher potential and is represented in the circuit symbol by the longer line.

![Circuit Symbols](image)

Figure 26.3

Parallel Combination

Two capacitors connected as shown in Figure 26.4a are known as a parallel combination of capacitors. Figure 26.4b shows a circuit diagram for this combination of capacitors. The left plates of the capacitors are connected to the positive terminal of the battery by a conducting wire and are therefore both at the same electric potential as the positive terminal. Likewise, the right plates are connected to the negative terminal and so are both at the same potential as the negative terminal. Therefore, the individual potential differences across capacitors connected in parallel are the same and are equal to the potential difference applied across the combination. That is,

\[ \Delta V_1 = \Delta V_2 = \Delta V \]  \hspace{1cm} (26.4)

where \( \Delta V \) is the battery terminal voltage.

After the battery is attached to the circuit, the capacitors quickly reach their maximum charge. Let’s call the maximum charges on the two capacitors \( Q_1 \) and \( Q_2 \).
The total charge $Q_{tot}$ stored by the two capacitors is the sum of the charges on the individual capacitors:

$$Q = Q_1 + Q_2 \quad (26.5)$$

Figure 26.4

Suppose you wish to replace these two capacitors by one equivalent capacitor having a capacitance $C_{eq}$ as in Figure 26.4c. The effect this equivalent capacitor has on the circuit must be exactly the same as the effect of the combination of the two individual capacitors. That is, the equivalent capacitor must store charge $Q$ when connected to the battery. Figure 26.4c shows that the voltage across the equivalent capacitor is $\Delta V$ because the equivalent capacitor is connected directly across the battery terminals. Therefore, for the equivalent capacitor,

$$Q = C_{eq} \Delta V = C_1 \Delta V_1 + C_2 \Delta V_2$$

$$C_{eq} = C_1 + C_2$$

where we have canceled the voltages because they are all the same. If this treatment is extended to three or more capacitors connected in parallel, the equivalent capacitance is found to be

$$C_{eq} = C_1 + C_2 + ... \quad (26.6)$$

Therefore, the equivalent capacitance of a parallel combination of capacitors is (1) the algebraic sum of the individual capacitances and (2) greater than any of the individual capacitances.
Series Combination

Two capacitors connected as shown in Figure 26.5a and the equivalent circuit diagram in Figure 26.8b are known as a series combination of capacitors. The left plate of capacitor 1 and the right plate of capacitor 2 are connected to the terminals of a battery. The other two plates are connected to each other and to nothing else; hence, they form an isolated system that is initially uncharged and must continue to have zero net charge. To analyze this combination, let’s first consider the uncharged capacitors and then follow what happens immediately after a battery is connected to the circuit. When the battery is connected, electrons are transferred out of the left plate of $C_1$ and into the right plate of $C_2$. As this negative charge accumulates on the right plate of $C_2$, an equivalent amount of negative charge is forced off the left plate of $C_2$, and this left plate therefore has an excess positive charge. The negative charge leaving the left plate of $C_2$ causes negative charges to accumulate on the right plate of $C_1$. As a result, both right plates end up with a charge $-Q$ and both left plates end up with a charge $+Q$. Therefore, the charges on capacitors connected in series are the same:

$$Q_1 = Q_2 = Q$$

(26.7)

where $Q$ is the charge that moved between a wire and the connected outside plate of one of the capacitors.

Figure 26.5a shows the individual voltages $\Delta V_1$ and $\Delta V_2$ across the capacitors. These voltages add to give the total voltage $\Delta V$ across the combination:

$$\Delta V = \Delta V_1 + \Delta V_2$$

(26.8)
Suppose the equivalent single capacitor in Figure 26.5c has the same effect on the circuit as the series combination when it is connected to the battery. Applying the definition of capacitance to the circuit in Figure 26.5c gives

$$\frac{Q}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

Canceling the charges because they are all the same gives

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

When this analysis is applied to three or more capacitors connected in series, the relationship for the equivalent capacitance is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ...$$  \hspace{1cm} (26.9)

**Quick Quiz 26.3.1**

Two capacitors are identical. They can be connected in series or in parallel. If you want the smallest equivalent capacitance for the combination,

A. do you connect them in series,
B. do you connect them in parallel,
C. do the combinations have the same capacitance?

**Quick Quiz 26.3.2**

A battery is attached to several different capacitors connected in parallel. Which of the following statements is true?

A. All capacitors have the same charge, and the equivalent capacitance is greater than the capacitance of any of the capacitors in the group.
B. The capacitor with the largest capacitance carries the smallest charge.
C. The potential difference across each capacitor is the same, and the equivalent capacitance is greater than any of the capacitors in the group.
D. The capacitor with the smallest capacitance carries the largest charge.

**Problem 26.3.1**

Two capacitors, $C_1 = 2.00 \ \mu F$ and $C_2 = 3.0 \ \mu F$, are connected in series, and the resulting combination is connected to a 9.00-V battery. Find

a) the equivalent capacitance of the combination.
b) the potential difference across each capacitor.
c) the charge stored on each capacitor.
26.4 Energy Stored in a Charged Capacitor

Because positive and negative charges are separated in the system of two conductors in a capacitor, electric potential energy is stored in the system. Many of those who work with electronic equipment have at some time verified that a capacitor can store energy. If the plates of a charged capacitor are connected by a conductor such as a wire, charge moves between each plate and its connecting wire until the capacitor is uncharged. The discharge can often be observed as a visible spark. If you accidentally touch the opposite plates of a charged capacitor, your fingers act as a pathway for discharge and the result is an electric shock. The degree of shock you receive depends on the capacitance and the voltage applied to the capacitor. Such a shock could be dangerous if high voltages are present as in the power supply of a home theater system. Because the charges can be stored in a capacitor even when the system is turned off, unplugging the system does not make it safe to open the case and touch the components inside.

Suppose \( q \) is the charge on the capacitor at some instant during the charging process. At the same instant, the potential difference across the capacitor is \( \Delta V = \frac{q}{C} \). From Section 25.1, we know that the work necessary to transfer an increment of charge \( dq \) from the plate carrying charge \(-q\) to the plate carrying charge \( q \) (which is at the higher electric potential) is

\[
dW = \Delta V dq = \frac{q}{C} dq
\]

The total work required to charge the capacitor from \( q = 0 \) to some final charge \( q = Q \) is

\[
W = \int_0^Q \frac{q}{C} dq = \frac{1}{C} \int_0^Q q dq = \frac{Q^2}{2C}
\]

The work done in charging the capacitor appears as electric potential energy \( U_E \) stored in the capacitor. Using Equation 26.1, we can express the potential energy stored in a charged capacitor as

\[
U_E = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} C \Delta V^2 = \frac{1}{2} Q \Delta V \quad (26.10)
\]

Equation 26.10 applies to any capacitor, regardless of its geometry. For a given capacitance, the stored energy increases as the charge and the potential difference increase. In practice, there is a limit to the maximum energy (or charge) that can be stored because, at a sufficiently large value of \( \Delta V \), discharge ultimately
occurs between the plates. For this reason, capacitors are usually labeled with a maximum operating voltage.

We can consider the energy in a capacitor to be stored in the electric field created between the plates as the capacitor is charged. This description is reasonable because the electric field is proportional to the charge on the capacitor. For a parallel-plate capacitor, the potential difference is related to the electric field through the relationship $\Delta V = Ed$. Furthermore, its capacitance is $C = \varepsilon_0 A/d$. Substituting these expressions into Equation 26.10 gives

$$U_e = \frac{1}{2} \varepsilon_0 A(Ed)^2 = \frac{1}{2} \varepsilon_0 (Ad)E^2$$

Because the volume occupied by the electric field is $Ad$, the energy per unit volume, known as the energy density, is

$$u_e = \frac{1}{2} \varepsilon_0 E^2$$

Quick Quiz 26.4.1

A parallel-plate capacitor is connected to a battery. What happens to the stored energy if the plate separation is doubled while the capacitor remains connected to the battery?
A. It remains the same.
B. It is doubled.
C. It decreases by a factor of 2.
D. It increases by a factor of 4.

Quick Quiz 26.4.2

A parallel-plate capacitor is charged and then is disconnected from the battery. By what factor does the stored energy change when the plate separation is then doubled?
A. It becomes four times larger.
B. It becomes two times larger.
C. It stays the same.
D. It becomes one-half as large.

26.5 Capacitors with Dielectrics

A dielectric is a nonconducting material such as rubber, glass, or waxed paper. We can perform the following experiment to illustrate the effect of a dielectric in a capacitor. Consider a parallel-plate capacitor that without a dielectric
has a charge $Q_0$ and a capacitance $C_0$. The potential difference across the capacitor is $\Delta V_0 = Q_0/C_0$. Figure 26.6a illustrates this situation. The potential difference is measured by a device called a voltmeter. Notice that no battery is shown in the figure; also, we must assume no charge can flow through an ideal voltmeter. Hence, there is no path by which charge can flow and alter the charge on the capacitor. If a dielectric is now inserted between the plates as in Figure 26.6b, the voltmeter indicates that the voltage between the plates decreases to a value $\Delta V$. The voltages with and without the dielectric are related by a factor $\varepsilon$ as follows:

$$\Delta V = \frac{\Delta V_0}{\varepsilon}$$

![Figure 26.6](image)

The dimensionless factor $\varepsilon$ is called the dielectric constant of the material. The dielectric constant varies from one material to another. In this section, we analyze this change in capacitance in terms of electrical parameters such as electric charge, electric field, and potential difference.

Because the charge $Q_0$ on the capacitor does not change, the capacitance must change to the value

$$C = \varepsilon C_0 \quad \text{(26.11)}$$

**Quick Quiz 26.5.1**

An electronics technician wishes to construct a parallel-plate capacitor using rutile (dielectric constant $\varepsilon=100$) as the dielectric. The area of the plates is 1.00 cm². What is the capacitance if the rutile thickness is 1.00 mm?
A. 88.5 pF
B. 177 pF
C. 100 µF
D. 35.4 µF
Chapter 27
CURRENT AND RESISTANCE

We now consider situations involving electric charges that are in motion through some region of space. We use the term electric current, or simply current, to describe the rate of flow of charge. Most practical applications of electricity deal with electric currents, including a variety of home appliances.

27.1 Electric Current

In this section, we study the flow of electric charges through a piece of material. The amount of flow depends on both the material through which the charges are passing and the potential difference across the material. Whenever there is a net flow of charge through some region, an electric current is said to exist.

To define current quantitatively, suppose charges are moving perpendicular to a surface of area A as shown in Figure 27.1. (This area could be the cross-sectional area of a wire, for example.) The current is defined as the rate at which charge flows through this surface. If ΔQ is the amount of charge that passes through this surface in a time interval Δt, the average current \( I_{\text{avg}} \) is equal to the charge that passes through A per unit time:

\[
I_{\text{avg}} = \frac{\Delta q}{\Delta t} \quad \text{(27.1)}
\]

If the rate at which charge flows varies in time, the current varies in time; we define the instantaneous current \( I \) as the limit of the average current as \( \Delta t \to 0 \):

\[
I = \frac{dq}{dt} \quad \text{(27.2)}
\]

The SI unit of current is the ampere (A): 1 A = 1 C/s.
The charged particles passing through the surface in Figure 27.1 can be positive, negative, or both. It is conventional to assign to the current the *same direction as the flow of positive charge*.

**Microscopic Model of Current**

We can relate current to the motion of the charge carriers by describing a microscopic model of conduction in a metal. Consider the current in a cylindrical conductor of cross-sectional area \( A \) (Fig. 27.2). The volume of a segment of the conductor of length \( \Delta x \) (between the two circular cross sections shown in Fig. 27.2) is \( A\Delta x \). If \( n \) represents the number of mobile charge carriers per unit volume (in other words, the charge carrier density), the number of carriers in the segment is \( nA\Delta x \). Therefore, the total charge \( \Delta Q \) in this segment is

\[
\Delta Q = (nA \Delta x)q
\]

where \( q \) is the charge on each carrier. If the carriers move with a velocity \( \bar{v}_d \) parallel to the axis of the cylinder, the magnitude of the displacement they experience in the \( x \) direction in a time interval \( \Delta t \) is \( \Delta x = v_d \Delta t \).

Dividing both sides of this equation by \( \Delta t \), we find that the average current in the conductor is

\[
I = nqv_dA
\]

**Quick Quiz 27.1.1**

Consider positive and negative charges moving horizontally through the four regions shown in Figure Q27.1.1. Rank the current in these four regions from highest to lowest.
Problem 27.1.1

The quantity of charge \( q \) (in coulombs) that has passed through a surface of area 2.00 cm\(^2\) varies with time according to the equation \( q = 4t^3 + 5t + 6 \), where \( t \) is in seconds. What is the instantaneous current through the surface at \( t = 1.00 \) s?

27.2 Resistance

Consider a conductor of cross-sectional area \( A \) carrying a current \( I \). The current density \( J \) in the conductor is defined as the current per unit area. Because the current \( I = nqv_dA \), the current density is

\[
J = \frac{I}{A} = nqv_d \quad (27.5)
\]

where \( J \) has SI units of amperes per meter squared. This expression is valid only if the current density is uniform and only if the surface of cross-sectional area \( A \) is perpendicular to the direction of the current.

A current density and an electric field are established in a conductor whenever a potential difference is maintained across the conductor. In some materials, the current density is proportional to the electric field:

\[
J = \sigma E \quad (27.6)
\]

where the constant of proportionality \( \sigma \) is called the conductivity of the conductor. Materials that obey Equation 27.6 are said to follow Ohm’s law, named after Georg Simon Ohm.
We can obtain an equation useful in practical applications by considering a segment of straight wire of uniform cross-sectional area $A$ and length $\ell$, as shown in Figure 27.3. A potential difference $\Delta V$ is maintained across the wire, creating in the wire an electric field and a current. If the field is assumed to be uniform, the magnitude of the potential difference across the wire is related to the field within the wire through equation $\Delta V = E\ell$. Therefore, we can express the current density in the wire as

$$J = \sigma \frac{\Delta V}{\ell}$$

Because $J = I/A$, the potential difference across the wire is

$$\Delta V = \frac{\ell}{\sigma} J = \left(\frac{\ell}{\sigma A}\right) I = RI$$

The quantity $R = \frac{\ell}{\sigma A}$ is called the resistance of the conductor. We define the resistance as the ratio of the potential difference across a conductor to the current in the conductor:

$$R = \frac{\Delta V}{I} \quad (27.7)$$

We will use this equation again and again when studying electric circuits. This result shows that resistance has SI units of volts per ampere. One volt per ampere is defined to be one ohm ($\Omega$).

The inverse of conductivity is resistivity $\rho$:

$$\rho = \frac{1}{\sigma}$$

where $\rho$ has the units ohm.meters ($\Omega$. m). Because $R = \frac{\ell}{\sigma A}$, we can express the resistance of a uniform block of material along the length $\ell$ as
Every ohmic material has a characteristic resistivity that depends on the properties of the material and on temperature.

**Quick Quiz 27.2.1**

A cylindrical wire has a radius \( r \) and length \( \ell \). If both \( r \) and \( \ell \) are doubled, the resistance of the wire

A. increases
B. decreases
C. remains the same.

**Quick Quiz 27.2.2**

Two wires A and B with circular cross sections are made of the same metal and have equal lengths, but the resistance of wire A is three times greater than that of wire B. What is the ratio of the radius of A to that of B?

**Problem 27.2.1**

A metal wire of resistance \( R \) is cut into three equal pieces that are then placed together side by side to form a new cable with a length equal to one-third the original length. What is the resistance of this new cable?

**Problem 27.2.2**

A resistor is constructed of a carbon rod (resistivity \( \rho = 3.5 \times 10^{-5} \)) that has a uniform cross-sectional area of 5.00 mm\(^2\). When a potential difference of 15.0 V is applied across the ends of the rod, the rod carries a current of \( 4.00 \times 10^{-3} \) A. Find

a) the resistance of the rod,

b) the rod’s length.

**27.3 Resistance and Temperature**

Over a limited temperature range, the resistivity of a conductor varies approximately linearly with temperature according to the expression

\[
\rho = \rho_0 (1 + \alpha (T - T_0))
\]  

(27.9)
where $\rho$ is the resistivity at some temperature $T$ (in degrees Celsius), $\rho_0$ is the resistivity at some reference temperature $T_0$ (usually taken to be 20°C), and $\alpha$ is the temperature coefficient of resistivity.

Because resistance is proportional to resistivity, the variation of resistance of a sample is

$$R = R_0(1 + \alpha(T - T_0))$$

(27.10)

where $R_0$ is the resistance at temperature $T_0$. Use of this property enables precise temperature measurements through careful monitoring of the resistance of a probe made from a particular material.

**Problem 27.3.1**

A metal wire has a resistance of 10.0 Ω at a temperature of 20.0°C. If the same wire has a resistance of 10.6 Ω at 90.0°C, what is the resistance of this wire when its temperature is -20.0°C?

**27.4 Superconductors**

There is a class of metals and compounds whose resistance decreases to zero when they are below a certain temperature $T_c$, known as the critical temperature. These materials are known as superconductors. The resistance–temperature graph for a superconductor follows that of a normal metal at temperatures above $T_c$ (Fig. 27.4). When the temperature is at or below $T_c$, the resistivity drops suddenly to zero. This phenomenon was discovered in 1911 by Dutch physicist Heike Kamerlingh-Onnes (1853–1926) as he worked with mercury, which is a superconductor below 4.2 K. Measurements have shown that the resistivities of superconductors below their $T_c$ values are less than $4 \times 10^{-25}$ Ωm, or approximately $10^{17}$ times smaller than the resistivity of copper. In practice, these resistivities are considered to be zero.
Today, thousands of superconductors are known, the critical temperatures of recently discovered superconductors are substantially higher than initially thought possible. Two kinds of superconductors are recognized. The more recently identified ones are essentially ceramics with high critical temperatures, whereas superconducting materials such as those observed by Kamerlingh-Onnes are metals. If a room-temperature superconductor is ever identified, its effect on technology could be tremendous.

The value of $T_c$ is sensitive to chemical composition, pressure, and molecular structure. Copper, silver, and gold, which are excellent conductors, do not exhibit superconductivity.

One truly remarkable feature of superconductors is that once a current is set up in them, it persists without any applied potential difference (because $R = 0$). Steady currents have been observed to persist in superconducting loops for several years with no apparent decay!

### 27.5 Electrical Power

Imagine following a positive quantity of charge $Q$ moving clockwise around the circuit in Figure 27.5 from point a through the battery and resistor back to point a. We identify the entire circuit as our system. As the charge moves from a to b through the battery, the electric potential energy of the system increases by an amount $Q\Delta V$ while the chemical potential energy in the battery decreases by the same amount. As the charge moves from c to d through the resistor, however, the electric potential energy of the system decreases due to collisions of electrons with...
atoms in the resistor. In this process, the electric potential energy is transformed to
internal energy corresponding to increased vibrational motion of the atoms in the
resistor. Because the resistance of the interconnecting wires is neglected, no energy
transformation occurs for paths bc and da. When the charge returns to point a, the
net result is that some of the chemical potential energy in the battery has been
delivered to the resistor and resides in the resistor as internal energy $E_{\text{int}}$ associated
with molecular vibration.

Figure 27.5

Let’s now investigate the rate at which the electric potential energy of the
system decreases as the charge $Q$ passes through the resistor

$$\frac{dU}{dt} = \frac{d}{dt} (Q\Delta V) = \frac{dQ}{dt} \Delta V = I \Delta V$$

where $I$ is the current in the circuit. The system regains this potential energy when
the charge passes through the battery, at the expense of chemical energy in the
battery. The rate at which the potential energy of the system decreases as the charge
passes through the resistor is equal to the rate at which the system gains internal
energy in the resistor. Therefore, the power $P$, representing the rate at which energy
is delivered to the resistor, is

$$P = I \Delta V \quad (27.11)$$

We derived this result by considering a battery delivering energy to a
resistor. Equation 27.11, however, can be used to calculate the power delivered by a
voltage source to any device carrying a current $I$ and having a potential difference
$\Delta V$ between its terminals.

Using equation 27.11 and $\Delta V = IR$ for a resistor, we can express the power
delivered to the resistor in the alternative forms

$$P = I^2 R = \frac{\Delta V^2}{R} \quad (27.12)$$
When I is expressed in amperes, $\Delta V$ in volts, and $R$ in ohms, the SI unit of power is the watt. The process by which energy is transformed to internal energy in a conductor of resistance $R$ is often called joule heating; this transformation is also often referred to as an $I^2R$ loss.

**Quick Quiz 27.5.1**

The same potential difference is applied to the two lightbulbs. Which one of the following statements is true?

A. The 30-W bulb carries the greater current and has the higher resistance.

B. The 30-W bulb carries the greater current, but the 60-W bulb has the higher resistance.

C. The 30-W bulb has the higher resistance, but the 60-W bulb carries the greater current.

D. The 60-W bulb carries the greater current and has the higher resistance.

![Figure Q27.5.1](image)

**Quick Quiz 27.5.2**

Two lightbulbs both operate on 120 V. One has a power of 25 W and the other 100 W. Which lightbulb carries more current?

A. The dim 25-W lightbulb does.

B. The bright 100-W lightbulb does.

C. Both are the same.

D. Not enough information is given to answer the question.
Problem 27.5.1

Two conductors made of the same material are connected across the same potential difference. Conductor A has twice the diameter and twice the length of conductor B. What is the ratio of the power delivered to A to the power delivered to B?

Problem 27.5.2

Assuming the cost of energy from the electric company is $0.110/kWh, compute the cost per day of operating a lamp that draws a current of 1.70 A from a 110-V line.
Chapter 28
DIRECT CURRENT CIRCUITS

In this chapter, we analyze simple electric circuits that contain batteries, resistors, and capacitors in various combinations. Some circuits contain resistors that can be combined using simple rules. The analysis of more complicated circuits is simplified using Kirchhoff’s rules, which follow from the laws of conservation of energy and conservation of electric charge for isolated systems. Most of the circuits analyzed are assumed to be in steady state, which means that currents in the circuit are constant in magnitude and direction. A current that is constant in direction is called a direct current (DC).

28.1 Electromotive Force

We will generally use a battery as a source of energy for circuits in our discussion. Because the potential difference at the battery terminals is constant in a particular circuit, the current in the circuit is constant in magnitude and direction and is called direct current. A battery is called either a source of electromotive force or, more commonly, a source of emf. The emf \( \mathcal{E} \) of a battery is the maximum possible voltage the battery can provide between its terminals. You can think of a source of emf as a “charge pump”. When an electric potential difference exists between two points, the source moves charges “uphill” from the lower potential to the higher.

We shall generally assume the connecting wires in a circuit have no resistance. The positive terminal of a battery is at a higher potential than the negative terminal. Because a real battery is made of matter, there is resistance to the flow of charge within the battery. This resistance is called internal resistance \( r \).

![Figure 28.1](image)

Figure 28.1 shows that the terminal voltage \( \Delta V \) must equal the potential difference across the external resistance \( R \), often called the load resistance. The
Load resistor might be a simple resistive circuit element as in Figure 28.1, or it could be the resistance of some electrical device (such as a toaster, electric heater, or lightbulb) connected to the battery (or, in the case of household devices, to the wall outlet). The resistor represents a load on the battery because the battery must supply energy to operate the device containing the resistance.

\[ \varepsilon = RI + rI \quad (28.1) \]

\[ I = \frac{\varepsilon}{R + r} \quad (28.2) \]

Equation 28.2 shows that the current in this simple circuit depends on both the load resistance \( R \) external to the battery and the internal resistance \( r \). If \( R \) is much greater than \( r \), as it is in many real-world circuits, we can neglect \( r \).

Multiplying Equation 28.1 by the current \( I \) in the circuit gives

\[ \varepsilon I = I^2 R + I^2 r \quad (28.3) \]

Equation 28.3 indicates that because power \( P = I \Delta V \), the total power output \( I \varepsilon \) associated with the emf of the battery is delivered to the external load resistance in the amount \( I^2 R \) and to the internal resistance in the amount \( I^2 r \).

Quick Quiz 28.1.1

In order to maximize the percentage of the power that is delivered from a battery to a device, the internal resistance of the battery should be

A. as low as possible
B. as high as possible
C. The percentage does not depend on the internal resistance.

Example 28.1.1

A battery has an emf of 12.0 V and an internal resistance of 0.05 \( \Omega \). Its terminals are connected to a load resistance of 3.00 \( \Omega \).

a) Find the current in the circuit and the terminal voltage of the battery.

b) Calculate the power delivered to the load resistor, the power delivered to the internal resistance of the battery, and the power delivered by the battery.
Problem 28.1.1

A battery has an emf of 15.0 V. The terminal voltage of the battery is 11.6 V when it is delivering 20.0 W of power to an external load resistor $R$.

a) What is the value of $R$?

b) What is the internal resistance of the battery?

28.2 Resistors in Series and Parallel

When two or more resistors are connected together as are the incandescent lightbulbs in Figure 28.2a, they are said to be in a series combination. Figure 28.2b is the circuit diagram for the lightbulbs, shown as resistors, and the battery. What if you wanted to replace the series combination with a single resistor that would draw the same current from the battery? What would be its value? In a series connection, if an amount of charge $Q$ exits resistor $R_1$, charge $Q$ must also enter the second resistor $R_2$. Otherwise, charge would accumulate on the wire between the resistors. Therefore, the same amount of charge passes through both resistors in a given time interval and the currents are the same in both resistors:

$$ I = I_1 = I_2 $$(28.4)

where $I$ is the current leaving the battery, $I_1$ is the current in resistor $R_1$, and $I_2$ is the current in resistor $R_2$.

The potential difference applied across the series combination of resistors divides between the resistors. In Figure 28.2b, because the voltage drop from $a$ to $b$ equals $I_1R_1$ and the voltage drop from $b$ to $c$ equals $I_2R_2$, the voltage drop from $a$ to $c$ is

$$ \Delta V = \Delta V_1 + \Delta V_2 = I_1R_1 + I_2R_2 $$ (28.5)
The potential difference across the battery is also applied to the equivalent resistance $R_{eq}$ in Figure 28.2c:

$$\Delta V = IR_{eq}$$

Combining these equations for $\Delta V$ gives $R_{eq} = R_1 + R_2$.

The equivalent resistance of three or more resistors connected in series is:

$$R_{eq} = R_1 + R_2 + \ldots$$ (28.6)

This relationship indicates that the equivalent resistance of a series combination of resistors is the numerical sum of the individual resistances and is always greater than any individual resistance.

Now consider two resistors in a parallel combination as shown in Figure 28.3. As with the series combination, what is the value of the single resistor that could replace the combination and draw the same current from the battery? Notice that both resistors are connected directly across the terminals of the battery. Therefore, the potential differences across the resistors are the same:

$$\Delta V = \Delta V_1 = \Delta V_2$$ (28.7)

where $\Delta V$ is the terminal voltage of the battery.

When charges reach point a in Figure 28.5b, they split into two parts, with some going toward $R_1$ and the rest going toward $R_2$. A junction is any such point in a circuit where a current can split. This split results in less current in each individual resistor than the current leaving the battery. Because electric charge is conserved, the current $I$ that enters point a must equal the total current leaving that point:
\[ I = I_1 + I_2 = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} \quad (28.8) \]

The current in the equivalent resistance \( R_{eq} \) in Figure 28.3c is

\[ I = \frac{\Delta V}{R_{eq}} \]

where the equivalent resistance has the same effect on the circuit as the two resistors in parallel; that is, the equivalent resistance draws the same current I from the battery. Combining these equations for I, we see that the equivalent resistance of two resistors in parallel is given by

\[ \frac{\Delta V}{R_{eq}} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} \quad \rightarrow \quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \]

An extension of this analysis to three or more resistors in parallel gives

\[ \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots \quad (28.9) \]

This expression shows that the inverse of the equivalent resistance of two or more resistors in a parallel combination is equal to the sum of the inverses of the individual resistances. Furthermore, the equivalent resistance is always less than the smallest resistance in the group.

Household circuits are always wired such that the appliances are connected in parallel. Each device operates independently of the others so that if one is switched off, the others remain on. In addition, in this type of connection, all the devices operate on the same voltage.

**Quick Quiz 28.2.1**

The terminals of a battery are connected across two resistors in series. The resistances of the resistors are not the same. Which of the following statements are correct?

A. The resistor with the smaller resistance carries more current than the other resistor.
B. The current in each resistor is the same.
C. The potential difference across each resistor is the same.
D. The potential difference is greatest across the resistor closest to the positive terminal.
Quick Quiz 28.2.2

With the switch in the circuit of Figure Q28.2.1a closed, there is no current in R₂ because the current has an alternate zero-resistance path through the switch. There is current in R₁, and this current is measured with the ammeter (a device for measuring current) at the bottom of the circuit. If the switch is opened (Fig. Q28.2.1b), there is current in R₂. What happens to the reading on the ammeter when the switch is opened?

A. The reading goes up.
B. The reading goes down.
C. The reading does not change.

Quick Quiz 28.2.3

Several resistors are connected in parallel. Which of the following statements are correct?

A. The equivalent resistance is greater than any of the resistances in the group.
B. The equivalent resistance is less than any of the resistances in the group.
C. The equivalent resistance depends on the voltage applied across the group.
D. The equivalent resistance is equal to the sum of the resistances in the group.

Quick Quiz 28.2.4
A series circuit consists of three identical lamps connected to a battery as shown in Figure Q28.2.4. The switch S, originally open, is closed. What happens to the brightness of lamp C?

A. It increases.
B. It decreases somewhat.
C. It does not change.
D. It drops to zero.

**Problem 28.2.1**

![Figure 28.2.1](image)

a) Find the equivalent resistance between points $a$ and $b$ in Figure 28.2.1.

b) A potential difference of 34.0 V is applied between points $a$ and $b$. Calculate the current in each resistor.

**28.3 Kirchhoff’s Rules**

As we saw in the preceding section, combinations of resistors can be simplified and analyzed using the expression $\Delta V = IR$ and the rules for series and parallel combinations of resistors. Very often, however, it is not possible to reduce a circuit to a single loop using these rules. The procedure for analyzing more complex circuits is made possible by using the following two principles, called *Kirchhoff’s rules*.

**a. Junction rule.** The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction:

$$\sum I_{in} = \sum I_{out}$$  \hspace{1cm} (28.10)

**b. Loop rule.** The sum of the potential differences across all elements around any closed circuit loop must be zero:

$$\sum \Delta V = 0$$
Kirchhoff’s first rule is a statement of conservation of electric charge. All charges that enter a given point in a circuit must leave that point because charge cannot build up or disappear at a point.

Kirchhoff’s second rule follows from the law of conservation of energy for an isolated system. Let’s imagine moving a charge around a closed loop of a circuit. When the charge returns to the starting point, the charge–circuit system must have the same total energy as it had before the charge was moved. The sum of the increases in energy as the charge passes through some circuit elements must equal the sum of the decreases in energy as it passes through other elements. The potential energy of the system decreases whenever the charge moves through a potential drop \(-\text{IR}\) across a resistor or whenever it moves in the reverse direction through a source of emf. The potential energy increases whenever the charge passes through a battery from the negative terminal to the positive terminal.

**Example 28.3.1**

Find the currents \(I_1\), \(I_2\), and \(I_3\) in the circuit shown in Figure E28.3.1.

\[\mathcal{E}_1 = 6\text{V}; \quad \mathcal{E}_2 = 3\text{V}; \quad r_1 = r_2 = 1\Omega; \quad R = 2\Omega.\]

![Figure E28.3.1](image)

**Solution**

We arbitrarily choose the directions of the currents as labeled in Figure.

\[\text{Junction A: } I_1 + I_2 = I \quad (1)\]
Loop (1): \[ \mathcal{E}_1 - I_1 R_1 - IR = 0 \iff 6 - I_1 - 2I = 0 \quad (2) \]
Loop (2): \[ -\mathcal{E}_2 + I_2 R_2 + IR = 0 \iff -3 + I_2 + 2I = 0 \quad (3) \]

From (1), (2), (3) \[
\begin{cases}
I_1 = 2.4\text{A} \\
I_2 = -0.6\text{A} \\
I = 1.8\text{A}
\end{cases}
\]

The negative sign for \( I_2 \) indicates that the direction of the current is opposite the assumed direction.

**Problem 28.3.1**

Determine the current in each branch of the circuit shown in Figure P28.3.1.

![Figure P28.3.1](image)

**Problem 28.3.2**

For the circuit shown in Figure, calculate

a) The current in the 2.00-Ω resistor

b) The potential difference between points a and b.

![Figure](image)
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